NAG Toolbox for MATLAB

f08ae

1 Purpose

f08ae computes the QR factorization of a real m by n matrix.

2 Syntax

```
[a, tau, info] = f08ae(a, 'm', m, 'n', n)
```

3 Description

f08ae forms the QR factorization of an arbitrary rectangular real m by n matrix. No pivoting is performed.

If $m \geq n$, the factorization is given by:

$$A = Q\begin{pmatrix} R \\ 0 \end{pmatrix},$$

where R is an n by n upper triangular matrix and Q is an m by m orthogonal matrix. It is sometimes more convenient to write the factorization as

$$A = \begin{pmatrix} Q_1 & Q_2 \end{pmatrix}\begin{pmatrix} R \\ 0 \end{pmatrix},$$

which reduces to

$$A = Q_1 R,$$

where Q_1 consists of the first n columns of Q, and Q_2 the remaining $m - n$ columns.

If $m < n$, R is trapezoidal, and the factorization can be written

$$A = Q\begin{pmatrix} R_1 & R_2 \end{pmatrix},$$

where R_1 is upper triangular and R_2 is rectangular.

The matrix Q is not formed explicitly but is represented as a product of $\min(m, n)$ elementary reflectors (see the F08 Chapter Introduction for details). Functions are provided to work with Q in this representation (see Section 8).

Note also that for any $k < n$, the information returned in the first k columns of the array **a** represents a QR factorization of the first k columns of the original matrix A.

4 References

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **a**(**lda**,$*$) – **double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The m by n matrix A.

5.2 Optional Input Parameters

1: **m – int32 scalar**

Default: The first dimension of the array **a**.

m, the number of rows of the matrix A.

Constraint: $\mathbf{m} \geq 0$.

2: **n – int32 scalar**

Default: The second dimension of the array **a**.

n, the number of columns of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda, work, lwork

5.4 Output Parameters

1: **a(lda,$*$) – double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{m})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

If $m \geq n$, the elements below the diagonal are overwritten by details of the orthogonal matrix Q and the upper triangle contains the corresponding elements of the n by n upper triangular matrix R.

If $m < n$, the strictly lower triangular part contains details of the orthogonal matrix Q and the remaining elements are overwritten by the corresponding elements of the m by n upper trapezoidal matrix R.

2: **tau($*$) – double array**

Note: the dimension of the array **tau** must be at least $\max(1, \min(\mathbf{m}, \mathbf{n}))$.

further details of the orthogonal matrix Q.

3: **info – int32 scalar**

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **m**, 2: **n**, 3: **a**, 4: **lda**, 5: **tau**, 6: **work**, 7: **lwork**, 8: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

The computed factorization is the exact factorization of a nearby matrix $(A + E)$, where

$$\|E\|_2 = O(\epsilon)\|A\|_2,$$

and ϵ is the ***machine precision***.

8 Further Comments

The total number of floating-point operations is approximately $\frac{2}{3}n^2(3m - n)$ if $m \geq n$ or $\frac{2}{3}m^2(3n - m)$ if $m < n$.

To form the orthogonal matrix Q f08ae may be followed by a call to f08af:

```
[a, info] = f08af(a, tau, 'k', min(m,n));
```

but note that the second dimension of the array **a** must be at least **m**, which may be larger than was required by f08ae.

When $m \geq n$, it is often only the first n columns of Q that are required, and they may be formed by the call:

```
[a, info] = f08af(a, tau);
```

To apply Q to an arbitrary real rectangular matrix C, f08ae may be followed by a call to f08ag. For example,

```
[c, info] = f08ag('Left', 'Transpose', a, tau, c, 'k', min(m,n));
```

forms $C = Q^{\mathrm{T}}C$, where C is m by p.

To compute a QR factorization with column pivoting, use f08be.

The complex analogue of this function is f08as.

9 Example

```
a = [-0.57, -1.28, -0.39, 0.25;
     -1.93, 1.08, -0.31, -2.14;
     2.3, 0.24, 0.4, -0.35;
     -1.93, 0.64, -0.66, 0.08;
     0.15, 0.3, 0.15, -2.13;
     -0.02, 1.03, -1.43, 0.5];
[aOut, tau, info] = f08ae(a)
```

```
aOut =
    3.6177   -0.5566    0.8474    0.7460
    0.4609   -2.0281    0.5514    1.1700
   -0.5492   -0.0457    1.3745   -1.4105
    0.4609    0.2828    0.0044   -2.3755
   -0.0358    0.0796   -0.0773   -0.5214
    0.0048    0.3003    0.8017    0.2558
tau =
    1.1576
    1.6969
    1.2131
    1.4956
info =
           0
```